

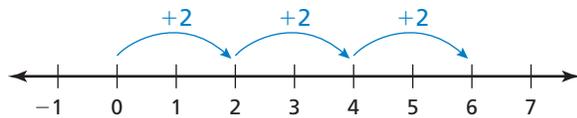
1.4 Multiplying Integers

Essential Question Is the product of two integers *positive*, *negative*, or *zero*? How can you tell?

1 EXAMPLE: Multiplying Integers with the Same Sign

Use repeated addition to find $3 \cdot 2$.

Recall that multiplication is repeated addition. $3 \cdot 2$ means to add 3 groups of 2.



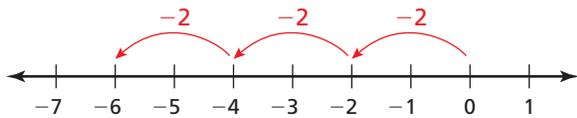
Now you can write
 $3 \cdot 2 = 2 + 2 + 2 = 6$.

So, $3 \cdot 2 = 6$.

2 EXAMPLE: Multiplying Integers with Different Signs

Use repeated addition to find $3 \cdot (-2)$.

$3 \cdot (-2)$ means to add 3 groups of -2 .



Now you can write
 $3 \cdot (-2) = (-2) + (-2) + (-2)$
 $= -6$.

So, $3 \cdot (-2) = -6$.

3 ACTIVITY: Multiplying Integers with Different Signs

Work with a partner. Use a table to find $-3 \cdot 2$.

Describe the pattern in the table. Use the pattern to complete the table.

2	•	2	=	4
1	•	2	=	2
0	•	2	=	0
-1	•	2	=	■
-2	•	2	=	■
-3	•	2	=	■

Notice the products decrease by 2 in each row.

So, continue the pattern.

$$-1 \cdot 2: 0 - 2 = \text{■}$$

$$-2 \cdot 2: -2 - 2 = \text{■}$$

$$-3 \cdot 2: -4 - 2 = \text{■}$$

So, $-3 \cdot 2 = \text{■}$.

4 ACTIVITY: Multiplying Integers with the Same Sign

Work with a partner. Use a table to find $-3 \cdot (-2)$.

Describe the pattern in the table. Use the pattern to complete the table.

-3	\cdot	3	$=$	-9
-3	\cdot	2	$=$	-6
-3	\cdot	1	$=$	-3
-3	\cdot	0	$=$	<input type="text"/>
-3	\cdot	-1	$=$	<input type="text"/>
-3	\cdot	-2	$=$	<input type="text"/>

Notice the products increase by 3 in each row.

So, continue the pattern.

$$-3 \cdot 0: -3 + 3 = \text{$$

$$-3 \cdot -1: 0 + 3 = \text{$$

$$-3 \cdot -2: 3 + 3 = \text{$$

So, $-3 \cdot (-2) = \text{$.

Inductive Reasoning

Work with a partner. Complete the table.

	Exercise	Type of Product	Product	Product: Positive or Negative
1	5. $3 \cdot 2$	Integers with the same sign		
2	6. $3 \cdot (-2)$	Integers with different signs		
3	7. $-3 \cdot 2$	Integers with different signs		
4	8. $-3 \cdot (-2)$	Integers with the same sign		
	9. $6 \cdot 3$			
	10. $2 \cdot (-5)$			
	11. $-6 \cdot 5$			
	12. $-5 \cdot (-3)$			

13. Write two integers whose product is 0.

What Is Your Answer?

14. **IN YOUR OWN WORDS** Is the product of two integers *positive*, *negative*, or *zero*? How can you tell?
15. Write general rules for multiplying (a) two integers with the same sign and (b) two integers with different signs.

Practice

Use what you learned about multiplying integers to complete Exercises 8–15 on page 26.

Key Ideas

Multiplying Integers with the Same Sign

Words The product of two integers with the same sign is positive.

Numbers $2 \cdot 3 = 6$ $-2 \cdot (-3) = 6$

Multiplying Integers with Different Signs

Words The product of two integers with different signs is negative.

Numbers $2 \cdot (-3) = -6$ $-2 \cdot 3 = -6$

EXAMPLE 1 Multiplying Integers with the Same Sign

Find $-5 \cdot (-6)$.

The integers have the same sign.

$$-5 \cdot (-6) = 30$$

The product is positive.

∴ The product is 30.

EXAMPLE 2 Multiplying Integers with Different Signs

Multiply.

a. $3(-4)$

b. $-7 \cdot 4$

The integers have different signs.

$$3(-4) = -12$$

$$-7 \cdot 4 = -28$$

The product is negative.

∴ The product is -12 .

∴ The product is -28 .

On Your Own

Multiply.

1. $5 \cdot 5$

2. $4(11)$

3. $-1(-9)$

4. $-7 \cdot (-8)$

5. $12 \cdot (-2)$

6. $4(-6)$

7. $-10(6)$

8. $-5 \cdot 7$

Now You're Ready
Exercises 8–23

EXAMPLE 3 Using Exponents

Study Tip

Place parentheses around a negative number to raise it to a power.

a. Evaluate $(-2)^2$.

$$\begin{aligned}(-2)^2 &= (-2) \cdot (-2) \\ &= 4\end{aligned}$$

Write $(-2)^2$ as repeated multiplication.
Multiply.

b. Evaluate -5^2 .

$$\begin{aligned}-5^2 &= -(5 \cdot 5) \\ &= -25\end{aligned}$$

Write 5^2 as repeated multiplication.
Multiply.

c. Evaluate $(-4)^3$.

$$\begin{aligned}(-4)^3 &= (-4) \cdot (-4) \cdot (-4) \\ &= 16 \cdot (-4) \\ &= -64\end{aligned}$$

Write $(-4)^3$ as repeated multiplication.
Multiply.
Multiply.

On Your Own

Evaluate the expression.

Now You're Ready
Exercises 32–37

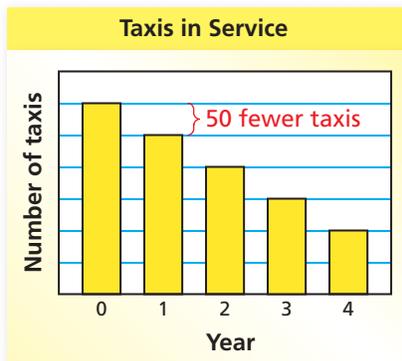
9. $(-3)^2$

10. $(-2)^3$

11. -7^2

12. -6^3

EXAMPLE 4 Real-Life Application



The bar graph shows the number of taxis a company has in service. The number of taxis decreases by the same amount each year for four years. Find the total change in the number of taxis.

The bar graph shows that the number of taxis in service decreases by 50 each year. Use a model to solve the problem.

$$\begin{aligned}\text{Total change} &= \text{Change per year} \cdot \text{Number of years} \\ &= -50 \cdot 4 \\ &= -200\end{aligned}$$

Use -50 for the change per year because the number decreases each year.

∴ The total change in the number of taxis is -200 .

On Your Own

13. A manatee population decreases by 15 manatees each year for 3 years. Find the total change in the manatee population.


Vocabulary and Concept Check

- WRITING** What do you know about the signs of two integers whose product is (a) positive and (b) negative?
- WRITING** How is $(-2)^2$ different from -2^2 ?

Tell whether the product is *positive* or *negative* without multiplying. Explain your reasoning.

- $4(-8)$
- $-5(-7)$
- $-3 \cdot (12)$

Tell whether the statement is *true* or *false*. Explain your reasoning.

- The product of three positive integers is positive.
- The product of three negative integers is positive.


Practice and Problem Solving

Multiply.

- $6 \cdot 4$
- $7(-3)$
- $-2(8)$
- $-3(-4)$
- $-6 \cdot 7$
- $3 \cdot 9$
- $8 \cdot (-5)$
- $-1 \cdot (-12)$
- $-5(10)$
- $-13(0)$
- $-9 \cdot 9$
- $15(-2)$
- $-10 \cdot 11$
- $-6 \cdot (-13)$
- $7(-14)$
- $-11 \cdot (-11)$

- JOGGING** You burn 10 calories each minute you jog. What integer represents the change in your calories after you jog for 20 minutes?
- WETLANDS** About 60,000 acres of wetlands are lost each year in the United States. What integer represents the change in wetlands after 4 years?

Multiply.

- $3 \cdot (-8) \cdot (-2)$
- $6(-9)(-1)$
- $-3(-5)(-4)$
- $-7(-3)(-5)$
- $-6 \cdot 3 \cdot (-6)$
- $3 \cdot (-12) \cdot 0$

Evaluate the expression.

- $(-4)^2$
- $(-1)^3$
- -8^2
- -6^2
- $-5^2 \cdot 4$
- $-2 \cdot (-3)^3$

ERROR ANALYSIS Describe and correct the error in evaluating the expression.

38.  $-2(-7) = -14$

39.  $-10^2 = 100$

ALGEBRA Evaluate the expression when $a = -2$, $b = 3$, and $c = -8$.

40. ab

41. $|a^2c|$

42. $ab^3 - ac$

NUMBER SENSE Find the next two numbers in the pattern.

43. $-12, 60, -300, 1500, \dots$

44. $7, -28, 112, -448, \dots$



45. **GYM CLASS** You lose four points each time you attend gym class without sneakers. You forget your sneakers three times. What integer represents the change in your points?

46. **AIRPLANE** The height of an airplane during a landing is given by $22,000 + (-480t)$, where t is the time in minutes.

- a. Copy and complete the table.
- b. Estimate how many minutes it takes the plane to land. Explain your reasoning.

Time	5 min	10 min	15 min	20 min
Height				

47. **INLINE SKATES** In June, the price of a pair of inline skates is \$165. The price changes each of the next three months.

- a. Copy and complete the table.

Month	Price of Skates
June	165 = \$165
July	$165 + (-12) = \$______$
August	$165 + 2(-12) = \$______$
September	$165 + 3(-12) = \$______$

- b. Describe the change in the price of the inline skates for each month.
- c. The table at the right shows the amount of money you save each month to buy the inline skates. Do you have enough money saved to buy the inline skates in August? September? Explain your reasoning.

Amount Saved	
June	\$35
July	\$55
August	\$45
September	\$18

48. **Reasoning** Two integers, a and b , have a product of 24. What is the least possible sum of a and b ?



Fair Game Review What you learned in previous grades & lessons

Divide.

49. $27 \div 9$

50. $48 \div 6$

51. $56 \div 4$

52. $153 \div 9$

53. **MULTIPLE CHOICE** What is the prime factorization of 84?

0

(A) $2^2 \times 3^2$

(B) $2^3 \times 7$

(C) $3^3 \times 7$

(D) $2^2 \times 3 \times 7$